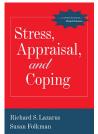


An Automata-Based Method to Formalize Psychological Theories : The Case Study of Lazarus and Folkman's Stress Theory

Gaspard Fougea, 2nd year PhD student at LMF under the direction of Alain Finkel and Stephane Le Roux ENS Paris-Saclay



Stress, Appraisal and Coping(Lazarus & Folkman, 1984)

Outline

- 1. Why formal models for psychology?
- 2. Why finite automata?
- 3. Finite automata and communicating automata
- 4. Transactional theory of stress (Lazarus & Folkman, 1984)
- 5. Modeling the transactional theory
- 6. Mathematical transformations for theory-building
- 7. Conclusion

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"We argue that formal theories provide this much needed set of tools, equipping researchers with tools for thinking, evaluating explanation, enhancing measurement, informing theory development, and promoting the collaborative construction of psychological theories." Robinaugh et al, 2021

What to expect from a type of formalism?

- 1. Openness to all psychological theories, both cognitive and behavioral,
- 2. Modularity (easy to modify, compose, and refine),
- 3. Having a formal dynamic,
- 4. Formal composition and refinement,
- 5. Capability to handle large systems,
- 6. Possibility of step-by-step simulation,
- 7. Formal verification of properties (psychological model checking) with the use of automatic tools,
- 8. Formal (and automatic) comparison of models, with automatic determination of compatibility between theories.

Why finite automata? What is verification?

- Identifying important properties with logical formulas or automata (specification)
- Mathematically verifying when these properties happen (bugs...)





Finite automata can be automatically verified!

Formal methods for psychology today : mostly statistics & probabilities.

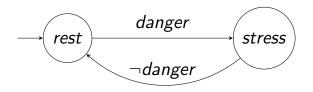


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- Formal methods for psychology today : mostly statistics & probabilities.
- In psychological theories: we naturally identify states and transitions
- Theoretical CS: powerful tools to model these theories. Finite automata & extensions!





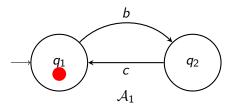
Basic understanding of stress

3. Finite automata and communicating automata

 $\mathcal{A} = (\Sigma, Q, \delta, I)$

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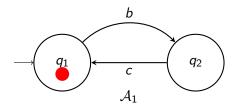




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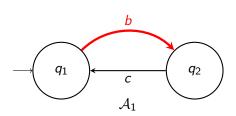
 Σ an alphabet Q set of states $\delta \subseteq Q \times \Sigma \times Q$ set of transitions $I \subseteq Q$ initial states





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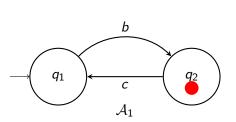
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Example:

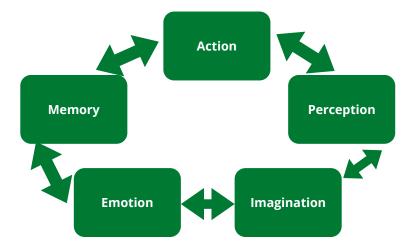
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Example:

N modules communicating \rightarrow N automata.

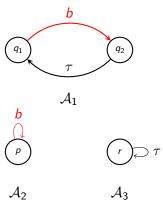


Synchronisation via multiple "handshake" (Millner, 1989; Horn & Sangnier, 2020; ...)

Set of automata synchronise on letters.

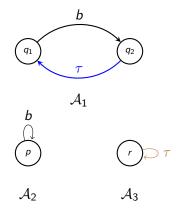
Example: System (A_1, A_2, A_3) synchronising on letter *b*

Rule: all automata with letter *b* must synchronise

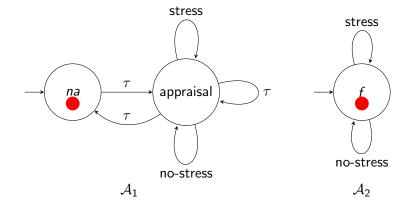


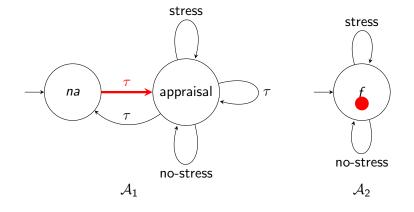
Meant to model imprecision "we don't know"

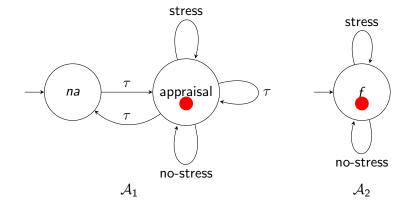
• τ -transitions don't synchronise with other automata

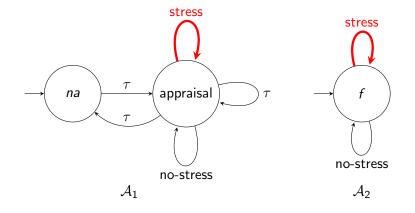


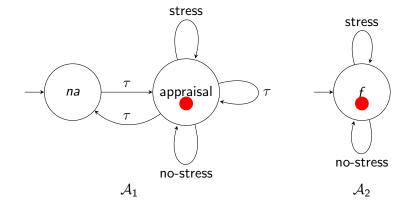
 τ -transition in A_1 and τ -transition in A_3 happen independently!









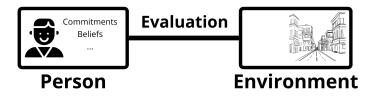




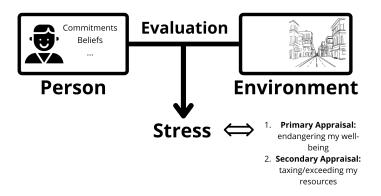




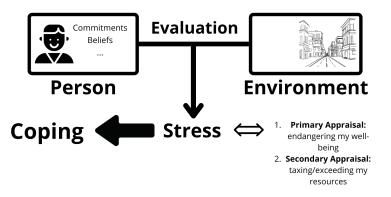
Environment



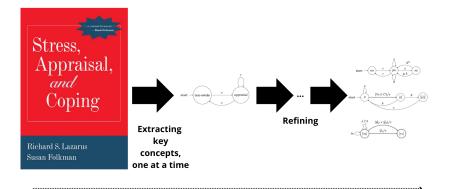
Appraisal= Evaluation



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5. Modeling the transactional theory

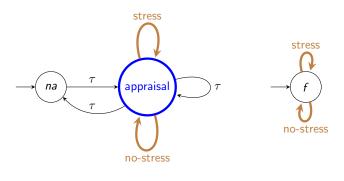


Verbal theory

Formal model

Modeling the transactional theory Modeling appraisal : System S_1

"Cognitive appraisal is an evaluative process that determines why and to what extent a particular transaction or series of transactions between the person and the environment is stressful." (Lazarus & Folkman, 1984)

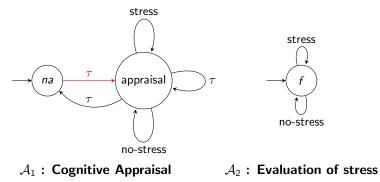


 \mathcal{A}_1 : Cognitive Appraisal

 \mathcal{A}_2 : Evaluation of stress

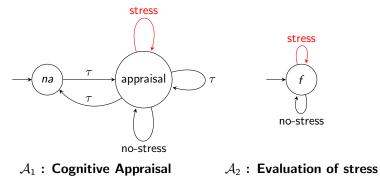
Modeling appraisal : System \mathcal{S}_1

A person who wakes up, appraises the person-environment relationship as stressful, appraises again this relationship as non-stressful, then goes back to sleep: $(non - awake, f) \xrightarrow{\tau} (appraisal, f) \xrightarrow{stress} (appraisal, f) \xrightarrow{no-stress} (appraisal, f) \xrightarrow{\tau} (non - awake, f)$



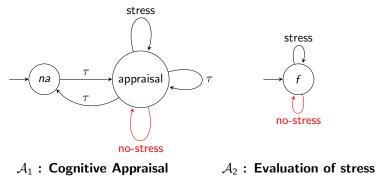
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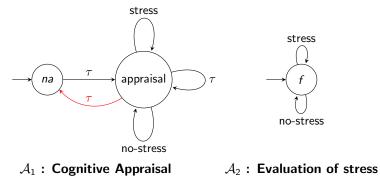
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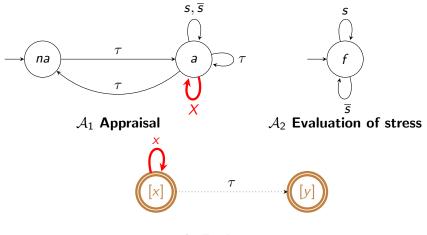


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Modeling the transactional theory Adding the environment: Refining S_1 into S_2

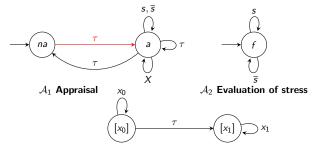


 \mathcal{A}_3 Environment

Adding the environment: Refining \mathcal{S}_1 into \mathcal{S}_2

A person who wakes up, perceives its environment, appraises the person-environment relationship as stressful, the environment changes by itself, the person perceives it, and appraises the new person-environment relationship as non-stressful.

$$(na, f, [x_0]) \xrightarrow{\tau} (a, f, [x_0]) \xrightarrow{x_0} (a, f, [x_0]) \xrightarrow{s} (a, f, [x_0]) \xrightarrow{\tau} (a, f, [x_1]) \xrightarrow{x_1} (a, f, [x_1]) \xrightarrow{\overline{s}} (a, f, [x_1])$$

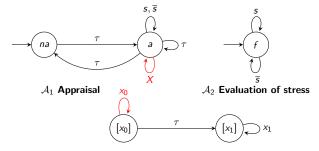


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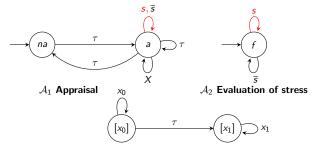


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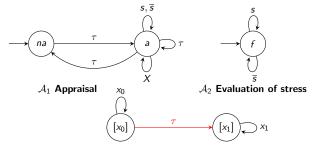


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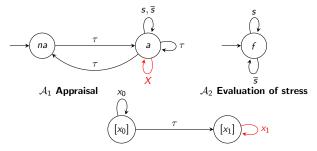


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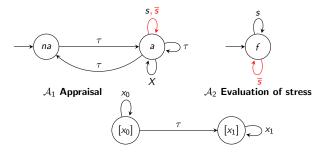
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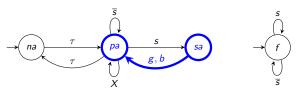
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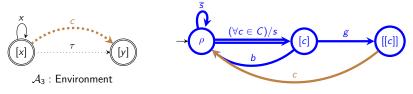


 \mathcal{A}_3 Environment



 \mathcal{A}_1 : Appraisal

 \mathcal{A}_2 : Calculating stress



 \mathcal{A}_4 : Secondary appraisal and coping

$$\begin{array}{l} (na, f, [x_0], \rho) \xrightarrow{\tau} (pa, f, [x_0], \rho) \xrightarrow{x_0} (pa, f, [x_0], \rho) \xrightarrow{s} \\ (sa, f, [x_0], [c_1]) \xrightarrow{b} (pa, f, [x_0], \rho) \xrightarrow{s} (sa, f, [x_0], [c_2]) \xrightarrow{g} \\ (sa, f, [x_0], [[c_2]]) \xrightarrow{c_2} (pa, f, [x_1], \rho) \xrightarrow{x_1} (pa, f, [x_1], \rho) \xrightarrow{\overline{s}} \\ (pa, f, [x_1], \rho). \end{array}$$

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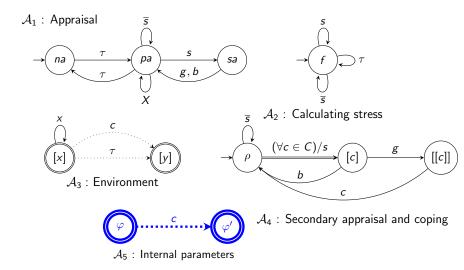
Modeling the transactional theory Adding commitments: refining S_3 into S_4

"Commitments express what is important to the person, what has meaning for him or her." (Lazarus & Folkman, 1984) Commitments: function

$$\varphi: X \to \{0,1\}$$

Person is stressed $\iff \varphi(x) = 0$ Some coping strategies affect commitments: φ can change.

Modeling the transactional theory Adding commitments: refining S_3 into S_4



Modeling the transactional theory **Example:**

Person's relationship with money: "It's important for me to have enough money. I want to feel like I'm safe financially. Enough money for me is having more than 1000 euros in my bank account. Right now I have enough money. Sometimes people steal money from my bank account and I have no money left. As a way to make myself feel better, I try to save a lot of money each month, and I try to think that money is not so important"

 $X=\{\geq 1000,<1000\},$ (more than 1000 euros or less than 1000 euros)

$$\varphi: X \to \{0,1\} \ \varphi(\geq 1000) = 1 \text{ and } \varphi(< 1000) = 0$$

$$\Phi = \{\varphi, \mathbf{1} - \varphi, \mathbf{1}, \mathbf{0}\}, \ (\mathbf{1} \text{ always 1}).$$

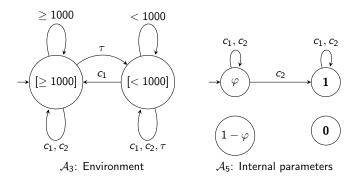
 c_1 : "saving money"

 c_2 : "trying to think money is not important" One τ -transition: "Sometimes people steal money from my bank account and I have no money left"

Modeling the transactional theory **Example:**

- c1 : "saving money"
- c_2 : "trying to think money is not important"

One $\tau\text{-transition:}$ "Sometimes people steal money from my bank account and I have no money let"



Modeling the transactional theory Adding more modules : $M_1, M_2, M_3...$

- M_1 : Imagining a course of actions and its results
- ► *M*₂ : Memorising coping strategies
- ► *M*₃ : Decision-making theories
- M₄ : Calculating the stress-level based on the goals, values and believes of the individual
- ► ...
- Any compatible cognitive theory!

6. Verification

Verification

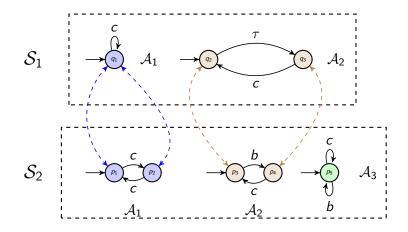
Modeling appraisal : System S_1

"Cognitive appraisal is an evaluative process that determines why and to what extent a particular transaction or series of transactions between the person and the environment is stressful." "Cognitive appraisal can be most readily understood as the process of categorizing an encounter, and its various facets, with respect to its significance for well-being. (...) Rather, it is largely evaluative, focused on meaning or significance, and takes place continuously during waking life."(Lazarus & Folkman, 1984)

$$G(a \Longleftrightarrow aw)$$
$$G(a \implies aW(na \lor s \lor \overline{s}))$$

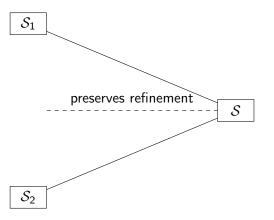
7. Mathematical transformations for theory-building

Mathematical transformations for theory-building **3 types of refinement**



Mathematical transformations for theory-building Merging: comparing and combining theories

- Preserves refinement
- When there is no common state: juxtaposition
- Otherwise, verifying compatibility and merging



8. Perspectives & conclusions

Perspectives & conclusions

- Further develop the methodology
- Formalise other theories: GWT, active inference, memory, appraisal theories...
- Formally compare different theories of the same field
- Create a big modular theory by merging different theories

Perspectives & conclusions

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